Dissipation in a Simple Model of a Topological Josephson Junction

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The topological features of low-dimensional superconductors have created a lot of excitement recently because of their broad range of applications in quantum information and their potential to reveal novel phases of quantum matter. A potential problem for practical applications is the presence of phase slips that break phase coherence. Dissipation in nontopological superconductors suppresses phase slips and can restore long-range order. Here, we investigate the role of dissipation in a topological Josephson junction. We show that the combined effects of topology and dissipation keep phase and antiphase slips strongly correlated so that the device is superconducting even under conditions where a nontopological device would be resistive. The resistive transition occurs at a critical value of the dissipation that is 4 times smaller than that expected for a conventional Josephson junction. We propose that this difference could be employed as a robust experimental signature of topological superconductivity.

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The study of topological materials, especially superconductors, is rapidly becoming a forefront research field for its potential to unveil novel forms of quantum matter and its relevance in quantum information applications [1–4]. The existence of superconductors with topological features was first speculated in $\nu = 5/2$ fractional quantum hall states [5] and then on the edges of effectively spinless systems with triplet pairing symmetry [6–8]. Later [9], it was proposed to realize topological superconductivity with surface states using the proximity effect between a strong topological insulator and an ordinary s-wave superconductor. Further work [2,10] has revealed that this requirement can be realized in one-dimensional semiconductor wires. Several other proposals have been put forward recently in order to experimentally observe topological superconductivity [11–14]. Of special interest in the following are the results of Ref. [15] in which it was found that the current phase relationship in thin film d-wave superconductors with an appropriate boundary geometry is strongly modified by subgap quasiparticle bound states.

The recent claimed observation [16,17] of Majorana fermions in InSb nanowires has further boosted interest in this problem. The experimental evidence of Majorana fermions is based on zero-bias conductance anomalies [16,17]. However, the relation between a zero-bias peak and the presence of a Majorana mode remains controversial [18], so robust experimental signatures of topological features must still be considered an open problem. Moreover, charging effects and quantum fluctuations, important in low-dimensional superconductors, must also be taken into account for a realistic comparison between theoretical models and experiments.

Here, we tackle both issues by studying a Josephson junction (JJ) composed of two topological superconductors separated by a weak link. Our model includes charging and dissipative effects that are relevant for experiments. Starting from a microscopic Hamiltonian, we show that dissipation in a topological JJ suppresses phase slips, induced by charging effects, more strongly than in a conventional JJ. We have identified a critical value of the dissipation strength, which is 4 times smaller than that in conventional JJs, above which phase slips are suppressed and a supercurrent is stable. We also propose an experimental setting in which this difference could be used as a robust signature of the existence of Majorana fermions.

We start with a brief introduction to the physics of conventional JJs. In bulk samples, the Josephson effects correspond to the existence of a current $I = I_c \sin(\phi)$ between two superconductors separated by a thin metal or insulator [8,19], where $\phi \equiv \phi_1 - \phi_2$ is the phase between the two superconductors and [20] $I_c \approx (\pi\Delta)/2R_N e$ is the so-called critical current with e the electron charge, Δ the zero temperature superconducting gap, and R_N the normal-state resistance.

As the system size decreases, charging effects induce fluctuations in the phase that can potentially destroy phase coherence. At the same time, there are different mechanisms of dissipation [21] that can quench these fluctuations and restore long-range order. In a certain region of parameters, it was found [22–24] that quasiparticle dissipation is equivalent to the one introduced by Caldeira and

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Leggett [21] to describe Ohmic dissipation in a quantum system induced by a linear coupling to a bath of harmonic oscillators $S_{\text{diss}}[\phi] = \eta/4\pi \int d\tau d\tau' \{[\phi(\tau) - \phi(\tau')]/(\tau - \tau')\}^2$, where in the JJ context $\eta = \hbar/4e^2 R_N \propto R_q/R_N$ and $R_q = h/4e^2$ is the quantum resistance. The effective action for the superconducting phase contains a 2π-periodic potential with degenerate minima. Tunneling among different minima, also referred to as a phase slip or instanton, lowers the ground-state energy and tends to delocalize the phase, weakening phase coherence. Ohmic dissipation, induced by the proximity to normal-state conducting channels, suppresses tunneling [21] and can help to restore superconductivity.

The interplay between these two mechanisms has been thoroughly investigated in the literature both for a double well [25,26] and for a periodic potential (sine-Gordon) [1,27,28]. In the limit where instantons are dilute, the partition function can be calculated by integrating over all multi-instanton paths. A renormalization group analysis of the resulting expression [29–34] confirms that, at zero temperature, there is a continuous phase transition at $\eta = \eta_c$ from a phase where phase slips destroy global superconductivity, to a superconducting phase where the order parameter stays in a single potential minimum. Dissipation introduces instanton-(anti-)instanton correlations which eventually fully suppress tunneling of the phase for $\eta \geq \eta_c$. A transition can only occur when these correlations are sufficiently long-range as in the case of Ohmic dissipation Ref. [21]. However, for intrinsic quasiparticle dissipation [22] the correlations are short range and, therefore, are not enough to stabilize global superconductivity. In that case the effect of dissipation is simply to weaken charging effects by renormalizing the capacitance. Phase slips will likely still create a local voltage fluctuation making the junction resistive. The ultimate reason for this behavior can be traced back to the energy gap 2Δ that severely penalizes quasiparticle tunneling at low temperatures.

The model.—In order to address nontrivial topological effects on the Josephson current, we consider two spinless superconducting p-wave chains coupled by a weak link. The simplest tight-binding model with these features is

$$H = \sum_{n=0,l=R,L} t(c^\dagger_{n,l} c_{n+1,l} + \text{H.c.}) + s(c^\dagger_{0,L} c_{0,R} + \text{H.c.}) - g \sum_{n,l} c^\dagger_{n+1,l} c_{n+1,l} c^\dagger_{n,l} c_{n,l}, \tag{1}$$

where t is the intrawire hopping, s is the weak link tunneling, and g is the effective coupling constant. At the mean-field level with $\Delta_{n,n+1;l} = -g\langle c_{n,l} c_{n+1,l}\rangle$, this Hamiltonian corresponds to a generalized Kitaev model [4] by the substitution $-g c^\dagger_{n+1,l} c_{n+1,l} c^\dagger_{n,l} c_{n,l} \to c^\dagger_{n+1,l} c^\dagger_{n,l} \Delta_{n,n+1;l} + \bar{\Delta}_{n,n+1;l} c_{n,l} c_{n+1,l} + g^{-1} \bar{\Delta}_{n,n+1;l} \Delta_{n,n+1;l}$. As in the nontopological case, the effective low-energy theory of the model involves only the difference between the superconducting phases across the weak link $\phi = \arg(\Delta_{0,1;L}) - \arg(\Delta_{0,1;R})$. Proposals to engineer such a model [35] considered a physical setup consisting of a one-dimensional wire where superconductivity is induced by proximity effect and topological features are a consequence of a strong spin-orbit coupling together with a perpendicularly applied magnetic field. The proximity to the nearby bulk superconductor induces an effective attractive density-density interaction between electrons on neighboring atomic sites. We refer to the recent review of Ref. [8] for an introduction to topological superconductivity and its possible experimental realization.

The microscopic derivation of the effective action for the junction from Eq. (1) follows the Eckern-Schoen-Ambegaokar calculation [22] for a conventional (nontopological) superconductor with an important difference: the presence of a bound state at the weak link. In the topological case, the single particle Green's function can be decomposed into a bound state and a continuum part. The former represents the effect of the gapped quasiparticles and, as in the nontopological case, can be treated in second-order perturbation theory in the weak link hopping magnitude s. This contribution yields an effective capacitive term, proportional to $(\partial_\tau \phi)^2$, and the Josephson term, proportional to $\cos(\phi)$ [22]. The bound-state contribution cannot be treated perturbatively and requires the knowledge of the bound-state wave function.

As the bound-state wave function cannot decay to the quasiparticle continuum, the occupation of the mixed particle-hole wave function—corresponding to two Majorana modes—is not a dynamic variable, being either empty or occupied. This problem has been considered by Pekker *et al.* [36] for the case where the magnitude of the order parameter equals the intrawire hopping $|\Delta| = t$, corresponding to a particularly simple form of the bound-state wave function. The appearance of a new $\cos[\phi(\tau)/2]$ term, particularly transparent in the treatment of Ref. [36], is expected to occur for all values of the intrawire hopping.

At zero temperature, after integration over the fermionic degrees of freedom, the effective Euclidean action is given by

$$S_{\text{eff}} = \int \left[\frac{(\partial_\tau \phi)^2}{16E_c} - E_J(1-\cos\phi) \pm \frac{E_M}{2}\cos(\phi/2)\right] d\tau', \tag{2}$$

which corresponds to the so-called double sine-Gordon action [36] where E_c is the charging energy due to the capacitance, which will eventually be renormalized by quasiparticle tunneling. E_J is the Josephson coupling, and E_M is the energy associated with the two Majorana fermions localized at the weak link that is proportional to the hopping amplitude s for an electron to tunnel across the junction. The positive (even) and negative (odd) energy states in this setup correspond to whether the bound state made of the two single Majorana fermions is occupied or empty ($n_b = 1, 0$). Here, parity corresponds to the eigenvalue of the number operator of the bound state [8]. This symmetry labels the two lowest energy states of the system. Note that (see Fig. 1) the different parities are related by a translation of the potential by 2π along the ϕ axis. In the following, without loss of generality, we only treat odd parity and infer the even-parity results from the translational symmetry. Defining $\mu = 8E_C E_J$, $\lambda = 4E_C E_M$, the double sine-Gordon potential (with $\lambda > 0$)

$$V(\phi) = \mu[1 - \cos(\phi)] + \lambda[1 - \cos(\phi/2)] \tag{3}$$

is shown schematically in Fig. 1 for two qualitatively different cases characterized by the existence or not of a local minimum.

We are assuming that the occupation number of the Majorana bound state is fixed. However, in a realistic situation, n_b is likely to fluctuate due to the coupling of the bound state to continuum states and inelastic processes [37]. Our derivation holds in the regime where the bound-state lifetime is larger than the characteristic time of the phase dynamics.

We now consider the role of a dissipative term in the topological junction. The total action is, thus, given by

$$S_{\rm top}[\phi] = \frac{1}{8E_C}(S_0[\phi] + S_{\rm diss}[\phi]), \tag{4}$$

where

$$S_0 = \int \left\{ \frac{(\partial_\tau \phi)^2}{2} - V[\phi] \right\} d\tau$$

and $S_{\rm diss}$ acquires the Caldeira and Leggett [21] form,

$$S_{\rm diss} = \tilde{\eta} \int \frac{[\phi(\tau) - \phi(\tau')]^2}{(\tau - \tau')^2} d\tau d\tau',$$

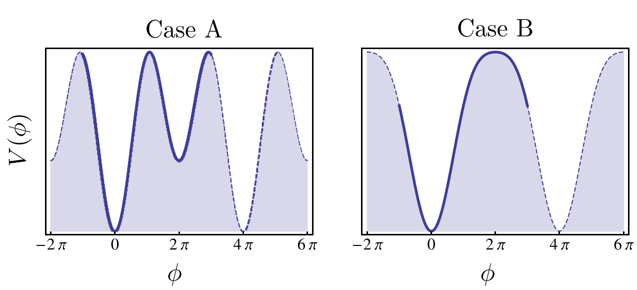

FIG. 1 (color online). Effective potential Eq. (2) for odd parity controlling the phase dynamic of a topological superconducting junction. Case A: $0 < \lambda < 4\mu$ and both a local and a global minimum exist. Case B: $\lambda > 4\mu$ and only a global minimum exists [38].

where $\tilde{\eta} = 8\eta E_c$. Note that for quasiparticle dissipation $\eta = \hbar/16\pi e^2 R_N$, while it is a free parameter for a generic resistive Ohmic shunt.

Results and discussion.—In this section, we carry out a saddle-point analysis of the action. The resulting field configurations, usually referred to as instantons, provide the leading-order contributions to the partition function in the semiclassical limit.

Depending on the ratio μ/λ, there are two qualitatively different configurations, depicted in Fig. 1, of the potential $V(\phi)$: case A, characterized by two local minima in the interval $[0, 4\pi)$, and case B, characterized by only one global minimum. The explicit solutions of the equation $\delta S_0 = 0$, found in Ref. [38], greatly simplify the theoretical analysis. Following Ref. [38], let us first discuss the bouncelike solution, existing only in case A, that starts and finishes at $\phi = 2\pi$. For the Wick rotated potential, shown in the left panel inset of Fig. 2, the bounce trajectory corresponds to the phase effectively rolling down the hill and bouncing back at a position where the potential equals that of the local minimum ($\phi = 2\pi$). This trajectory is given by

$$\phi_{\rm dsG} = \phi_{\rm sG}(\tau + R) + \phi_{\rm sG}[-(\tau - R)], \tag{5}$$

where $\phi_{\rm dsG}$ stands for the solution for the double sine-Gordon potential, $\phi_{\rm sG}(\tau) = 4\tan^{-1}[e^{m\tau}]$ is the instanton solution of the sine-Gordon model (i.e., the solution of the equations of motion with $\lambda = 0$), $R = (1/m)\sinh^{-1}[\sqrt{4\mu/\lambda - 1}]$ and $m^2 = \mu - \lambda/4$. These solutions are topologically trivial as they do not cause any phase slip, namely, the winding number of the phase after one bounce is still zero. In the context of quantum chromodynamics, it has been shown that these bounces contribute to tunneling but only perturbatively, so it is safe to neglect them with respect to the leading nonperturbative contribution to the action [39]. Moreover, ignoring these bouncing trajectories, enables a joint analysis of cases A and B.

Mussardo *et al.* [38] have also derived the classical instanton solution connecting the minima at $\phi = 0$ and $\phi = 4\pi$. This solution, shown schematically in the right panel of Fig. 2, is written as a superposition of sine-Gordon instantons,

$$\phi'_{\rm dsG}(\tau) = \phi_{\rm sG}(\tau + R') + \phi_{\rm sG}(\tau - R'), \tag{6}$$

with $R' = (1/m')\cosh^{-1}\sqrt{4\mu/\lambda + 1}$ and $m'^2 = \mu + \lambda/4$.

As shown in Fig. 2, the phase spends a time $2R'$ at the local minimum or maximum of the potential ($\phi = 2\pi$) before transitioning to the global minimum ($\phi = 4\pi$). The expression of Eq. (6) also gives the sine-Gordon instantons back in the limit of $\lambda \to 0$ for which $R' \to \infty$. This corresponds to the loss of the correlation between the two instantons in the double sine-Gordon solution. In this limit, therefore, the two sine-Gordon instantons can be regarded as free [38]. The results from Pekker *et al* [36] that

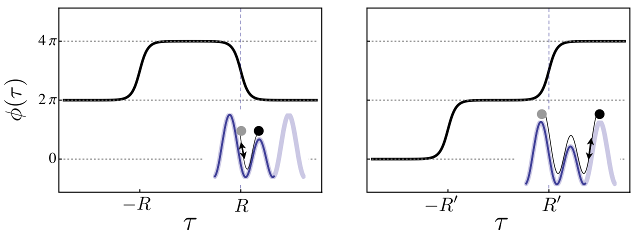

FIG. 2 (color online). Left: the bounce trajectory for case A. The solution is effectively the sum of an instanton and anti-instanton of the sine-Gordon model. Right: the trajectory of a single instanton for case A [38].

2π phase slips are suppressed can clearly be seen from the expression for ϕ'_{dsG} above, since $\phi'_{\mathrm{dsG}}(-\infty) = 0$ and $\phi'_{\mathrm{dsG}}(\infty) = 4\pi$.

Following the treatment of Schmid [28], we now assume the following approximate solution, valid in the dilute limit corresponding to large separations between instantons,

$$\Psi_{cl} = \sum_{j=1}^{n} e_j \phi'_{\mathrm{dsG}}(\tau - \tau_j), \tag{7}$$

where $e_j = +1(-1)$ for instantons (anti-instantons), n is the number of instantons or anti-instantons, and τ_j is the instanton's center of mass. The condition $\sum_{j=1}^{n} e_j = 0$ ensures that the action is finite. The proposed configuration corresponds to the leading-order contribution to the path integral in the limit in which phase slips are still rare events, and therefore, a linear superposition of well separated instantons is a good approximation to a typical configuration.

To begin the analysis of the instanton contribution to the action, we observe that within the dissipationless action there is no interaction between instantons since we have assumed the typical distance $|\tau_i - \tau_j|$ (with $i, j = 1, \ldots, n$) to be large. In this regime, the multi-instanton action can also be approximately given by the factorized expression $S_{0(n)} \approx n S_{0(1)}$ with $S_{0(1)}$ the action of a single instanton

$$S_{0(1)} = \int \left\{ \frac{1}{2} [\partial_\tau \phi'_{\mathrm{dsG}}(\tau)]^2 - V[\phi'_{\mathrm{dsG}}(\tau)] \right\} d\tau.$$

We now insert the solution above in the dissipative term of the action and make a further simplifying assumption, valid for large values of $m/\tilde{\eta}$: the instanton profile is replaced by a Heaviside θ function. After substituting this ansatz solution in the dissipative term of the action, integrating by parts twice, and neglecting second-order terms in R' (assuming $|\tau_i - \tau_j| \gg R'$), we obtain

$$S_{\mathrm{diss}} \approx 8\tilde{\eta}(2\pi)^2 \sum_{i,j}^{n} e_i e_j \log(\tau_i - \tau_j). \tag{8}$$

This result is identical to that obtained for a nontopological Josephson junction with Ohmic dissipation [1,28] except for the overall rescaling of the prefactor in the S_{diss} term. The theoretical analysis of Refs. [1,28], under the assumptions above, yields in our case a critical dissipation $\tilde{\eta}_c = E_C/4\pi^2$. When dissipation is induced by quasiparticle tunneling, the expression of $\tilde{\eta}_c$ above translates to

$$R_c = \frac{h}{e^2}, \tag{9}$$

where R_c is the critical normal-state resistance R_N. As a matter of comparison, the critical normal-state resistance for the nontopological Josephson junction $R_c = h/4e^2$ is 4 times less than that in the topological case.

We note that this result assumes that a small instanton fugacity $z = e^{-S_{0(1)}/(8E_C)} \ll 1$ has a negligible effect on $\tilde{\eta}_c$. Corrections to $\tilde{\eta}_c = E_C/4\pi^2$ due to a small z can still be computed systematically within the renormalization group framework of Refs. [1,28]. This correction, as for a nontopological JJ, slightly increases $\tilde{\eta}_c$ though its effect is relatively small in the dilute limit in which the instanton approach is applicable. Therefore, topological JJs are more robust to phase slips than the nontopological counterpart. A substantially smaller dissipation is sufficient to stabilize superconductivity in the topological case.

We note that for nontopological JJs, there is compelling experimental evidence [40], especially for $E_J \gg E_C$, that dissipation stabilizes superconductivity for $R_N < R_q$ with $R_q = h/4e^2$ the quantum resistance. The setup of Ref. [40] consisted of an Al-AlO_x-Al shunted tunnel junction of area $\sim 10^4\ \mathrm{nm}^2$ in which it is possible to tune the charging energy and the normal resistance. Measurements of the resistance $R \sim dI/dV$ versus I show qualitatively different behavior for $R_N < R_q$ and $R_N > R_q$. According to our theoretical prediction, the replacement of Al by a topological superconductor would change the value of the critical resistance so that we expect a transition not at $R_N = R_q$ but at $R_N = 4R_q$. Experimental observation of this change of behavior in the topological case would provide direct evidence of the presence of Majorana modes. The topological material could be an InSb [7] short semiconductor nanowire in contact with a bulk superconductor. Another promising material is the $LaAlO_3/SrTiO_3$ interface [41]. It is expected [42] that in a certain range of parameters this material is an intrinsic topological superconductor since the spin-orbit interaction is strong and can be tuned by the electric field effect. The one-dimensional step edges of the interface [43] are especially suited for a topological JJ setup.

In summary, we have studied the role of dissipation in a topological superconducting junction. In general, such a junction is more robust against fluctuations than the nontopological counterpart. The phase transition to a superconducting state occurs at a critical value of the dissipation that is 4 times smaller than that expected for a conventional Josephson junction. A tentative explanation for this difference is the existence of a single fermion (charge e)

topological current of Majorana fermions in addition to the conventional Cooper pairs (charge $2e$) supercurrent. We have proposed an experimental setup is which this difference could be used to find a robust signature of topological superconductivity.

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