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A comparative study of the $\Delta H(M,\Delta M)$ method reliability for square and triangular lattices

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The $\Delta H(M,\Delta M)$ method and its ability to determine the intrinsic switching field distributions of perpendicular recording media are numerically studied. Strong evidence is presented that this method, which is based on the mean-field approximation, has a well-defined reliability range, corroborating earlier results from micromagnetic and hysteron simulations. Specifically, it is shown that this well-behaved failure appears to be universal and independent from the actual lattice structure. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2828810]

I. INTRODUCTION

One of the most crucial properties of magnetic recording media is the intrinsic switching-field distribution (ISFD) $D(H_S)$ of the media grains because it is one of the key factors defining the recording quality.[1] Each media grain is characterized by an intrinsic switching field H_S as a local material property. Due to the fact that grains interact with each other by means of dipolar and exchange interactions, the distribution of these local fields $D(H_S)$ is not easily accessible in macroscopic measurements. Over the years, several attempts of varying success have been made to determine $D(H_S)$ from macroscopic magnetization-reversal-type measurements.[2–8] The recently developed $\Delta H(M,\Delta M)$ method has been used successfully in analyzing and quantifying progress in perpendicular recording media fabrication.[5,6,8] The method itself is a generalization of an earlier measurement technique, the ΔH_C method,[2] and has several advantages over comparable methods. Specifically, it allows for the determination of the entire $D(H_S)$ distribution and its functional form and not just a single characteristic parameter. Furthermore, it enables oversampling, which makes consistency checks feasible and gives one the opportunity to quantify the confidence level of its results.

In this paper, we numerically study the reliability of the $\Delta H(M,\Delta M)$ method. We model each media grain as a symmetric hysteron, which generates a rectangular hysteresis loop in an applied field H. The ferromagnetic layer system is then represented by a triangular lattice of symmetric hysterons with periodic boundary conditions applied to ensure that all hysterons are equivalent (see Fig. 1). The hysterons interact ferromagnetically with their six nearest neighbors with strength J and have an ISFD $D(H_S)$. Using a previously developed and described algorithm,[9] the major hysteresis loop and recoil curves for the symmetric hysteron model can be calculated. We present strong evidence that the $\Delta H(M,\Delta M)$ method fails in a well-defined way, corroborating earlier results.[6,9] Moreover, our results indicate that this failure mode is a universal property of this method and independent of the detailed lattice structure.

II. $\Delta H(M,\Delta M)$ METHOD

Based on the mean-field approximation, the field difference ΔH between a recoil curve, which starts at a certain distance ΔM away from saturation, and the major hysteresis loop can be written as a function of the magnetization M on both curves

$$\Delta H(M,\Delta M) = I^{-1}\left(\frac{1-M}{2}\right) - I^{-1}\left(\frac{1-M-\Delta M}{2}\right), \tag{1}$$

where I^{-1} is the inverse of the integral $I(x)=\int_{-\infty}^{x} D(H_S)dH_S$. Within the framework of this method, ΔH is independent of the grain interaction, which allows for a direct experimental access to $D(H_S)$. For certain parametrized distribution functions, one can derive analytic expressions for ΔH. For simplicity, we assume here $D(H_S)$ to be represented by a Gaussian distribution of width σ, for which one derives

$$\Delta H_G(M,\Delta M) = \sqrt{2}\sigma[\mathrm{erf}^{-1}(M+\Delta M) - \mathrm{erf}^{-1}(M)]. \tag{2}$$

Details of this method and the analysis formalism have been described previously.[5,6,8,9]

In numerical simulations, the reliability range of the $\Delta H(M,\Delta M)$ method can now be checked with two types of independent measures: firstly, conventional quality measures for numerical fits such as the square of the multiple correlation coefficient (R^2) as well as the percentage difference (P_d) between the fitting result and the actual input parameter, which is obviously a known quantity in simulations; secondly, we were able to show that the mean-field approxima-

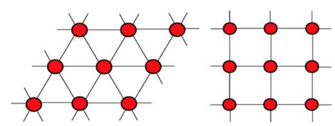

FIG. 1. (Color online) Triangular and square lattice in 2D.

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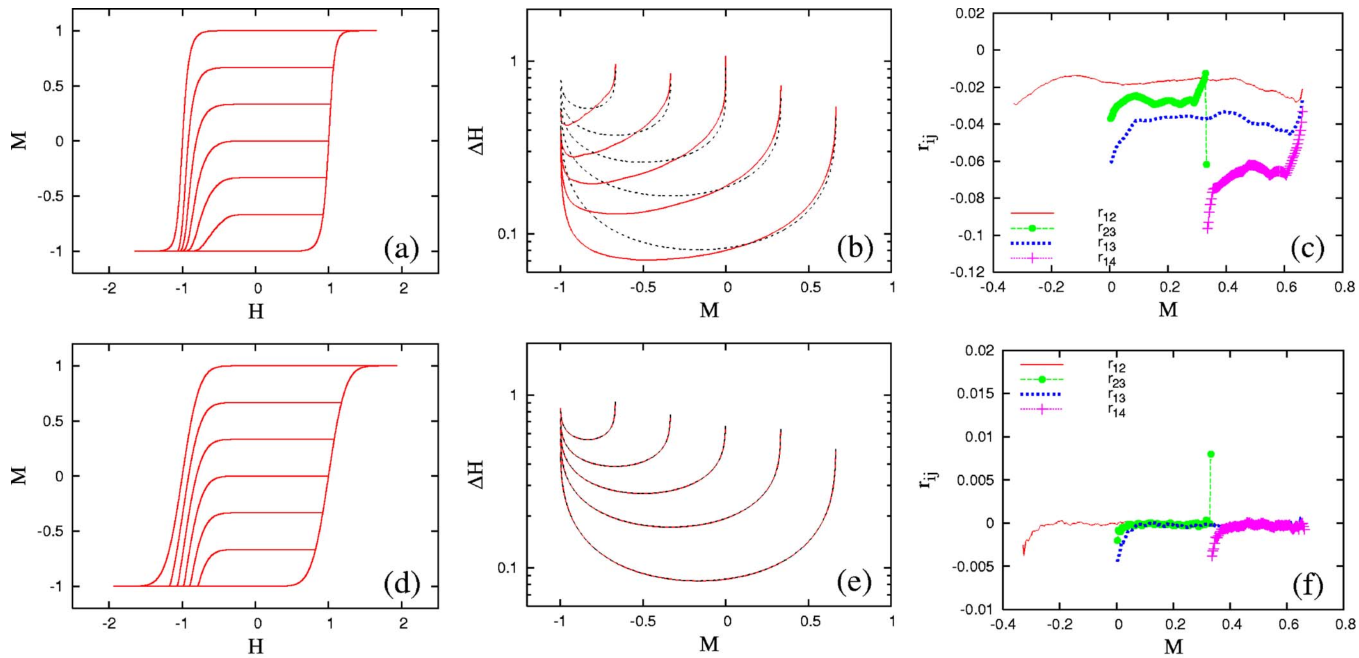

FIG. 2. (Color online) Numerical results for Gaussian ISFD on a 2D triangular lattice with 10^6 hysterons. Rows: (top) $\sigma/J=5$ and (bottom) $\sigma/J=50$. Columns: [Left, (a) and (d)] $M(H)$ curves, main loop, and five recoil curves. [Middle, (b) and (e)] $\Delta H(M,\Delta M)$ curves for the five recoil curves [(Solid lines) numerical result and (dotted lines) mean-field approximation]. [Right, (c) and (f)] Deviation from redundancy $[r_{ij}(M)]$ for the recoil curve pairs.

tion of this method causes redundancy in between multiple recoil curves.[9] So one can test data for deviations from this redundancy by means of a quantity $r=(1/n)\Sigma_{i,j}\langle r_{ij}^2(M)\rangle^{1/2}$, where $r_{ij}(M)$ is by definition identical to zero within the mean-field approximation[9] and n being the number of recoil curve pairs ij. The specific advantage of this reliability measure r is that it can be calculated from the data set alone, i.e., it is accessible from experimental data sets without the need for data fitting. We furthermore observed that r is rather robust against many numerical inaccuracies that are induced by the data fitting procedures itself, see, for instance, Ref. 9.

III. RESULTS AND DISCUSSION

To show that the $\Delta H(M,\Delta M)$ method fails reproducibly in a well-defined manner, we analyze the $\Delta H(M,\Delta M)$ data for Gaussian ISFD on a two-dimensional (2D) triangular lattice with 10^6 hysterons. Note that in our simulation, we set the ferromagnetic nearest-neighbor coupling strength $J=1$ and tune the distribution width σ because only the ratio σ/J is relevant for the reliability measure of the mean-field method. In this sense, a small (big) σ/J corresponds to strong (weak) nearest-neighbor exchange interactions of hysterons.

Key results of our numerical simulations are shown in Fig. 2. We show plots for $\sigma/J=5$ and 50 only to illustrate the general trends. Figure 2 displays the results for different σ/J's in different rows: (top) $\sigma/J=5$ and (bottom) $\sigma/J=50$. For each σ/J, we calculate a complete set of $M(H)$ curves, both the saturation hysteresis loop and recoil curves, as shown in the left column of Fig. 2. Note that M (or ΔM) is normalized to the saturation value M_S and H (or ΔH) is normalized to the coercive field H_C. In particular, we choose five equally spaced recoil curves, for which the distance to saturation is given by $\Delta M_i=i/3$ with i being an integer between 1 and 5. In the middle column of Fig. 2, we show the corresponding $\Delta H(M,\Delta M)$ curves (solid lines) derived from the simulated $M(H)$ curves, as well as the mean-field approximation of the $\Delta H(M,\Delta M)$ curves (dotted lines) calculated from Eq. (2). The mean-field curves are calculated by using the exact input parameter, which allows for a clear illustration of the deviations from mean-field behavior. From Figs. 2(b) and 2(e), we see that as σ/J increases and the role of exchange interaction decreases, the difference between the numerical result and the mean-field approximation becomes smaller. In the right column of Fig. 2, we show numerical values for the deviation from redundancy measures $r_{ij}(M)$ which are calculated from the simulated recoil curves shown in the left column. From Fig. 2(c), we see that for small σ/J, the deviation from data redundancy is quite substantial in the entire M definition range and for all the recoil curve pairs, while for high σ/J, the deviation is almost negligible for all M values and all recoil curve pairs, as one can see from Fig. 2(f).

The results here are very similar to the results obtained from a 2D square lattice with Gaussian ISFD.[9] To study the reliability range of the $\Delta H(M,\Delta M)$ method quantitatively, we plot the three previously mentioned reliability measures against the tuning parameter σ/J for both square and triangular lattices in Fig. 3.

Firstly, we find that those measures show very similar features for the two different lattices. All three quantities approach their mean-field approximation values with increasing σ/J (decreasing exchange interaction), i.e., $P_d\rightarrow 0$, $R^2\rightarrow 1$, and $r\rightarrow 0$ as $\sigma/J\rightarrow\infty$. As expected, the $\Delta H(M,\Delta M)$ method works very well for high σ/J in the sense that the fit quality is excellent and data redundancy is obtained.

Secondly, a clear shift of the failure toward higher σ/J

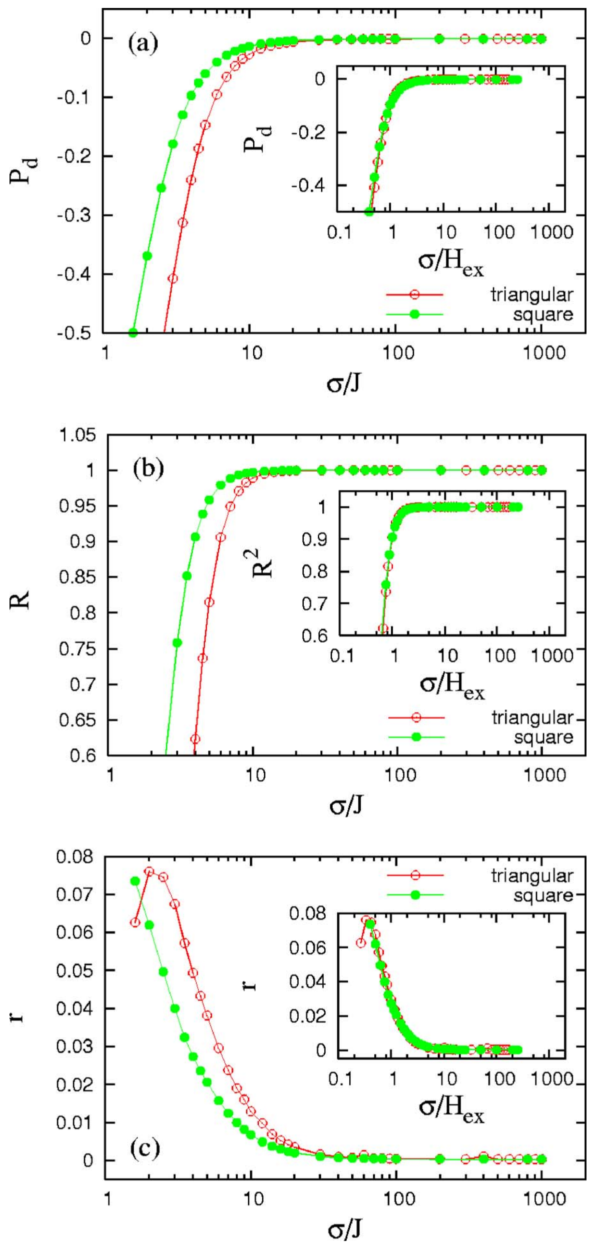

FIG. 3. (Color online) Reliability measures as functions of σ/J. (a) P_d, (b) R^2, and (c) r. Insets show the reliability measures as functions of σ/H_{ex}. All the calculations are done for Gaussian ISFD on 2D lattices with 10^6 hysterons.

values is seen in the reliability measures of the triangular lattice. To understand this shift, one has to keep in mind that the total exchange field H_{ex} to which every grain is exposed in triangular lattice is on average 50% higher than in square lattice, simply because the number of nearest neighbors n in triangular lattice is 6 instead of 4 (see Fig. 1). If we normalize for this effect (defining $H_{ex}=nJ$) by the saturated magnetization state, we find that the curves of the reliability measures against σ/H_{ex} for the two different lattices collapse onto each other very well, as can be seen from the insets in Fig. 3. This collapse strongly indicates that the deviation of the reliability measures as functions of σ/J is only due to the difference in the total aggregate exchange effect upon each lattice site.

The reliability range of the $\Delta H(M,\Delta M)$ method can be quantified by determining at which point one of the reliability measures becomes smaller (or greater) than a certain value. For example, we find that $R^2 \geqslant 0.98$ when $\sigma/H_{ex} \geqslant (\sigma/H_{ex})_c$. Here, we choose the measure R^2 and the certain value 0.98 specifically to compare it quantitatively with the earlier micromagnetic results.[6] In our calculations, we find $(\sigma/H_{ex})_c \simeq 1.5$ for both square and triangular lattices. In the original micromagnetic test,[6] it is found that the $\Delta H(M,\Delta M)$ method is still valid (in the sense that $R^2=0.98$) for $H_{ex}=0.21$ with $\sigma(H_k)=0.23$ [resulting in $\sigma(H_S)\simeq 0.26$] with all quantities given in units of the mean anisotropy field $\langle H_k \rangle$. This yields $(\sigma/H_{ex})_c \simeq 1.24$, which is in rough agreement with our result here. The difference could be related to the fact that the micromagnetic calculations included dipolar effects, which by themselves do not impact the reliability of the $\Delta H(M,\Delta M)$ method but cause a general broadening of magnetization curves. This could shift the onset of the exchange coupling caused failure toward lower σ values or higher levels of intergranular exchange coupling. However, this observed numerical difference could also be simply due to the noise level of the previously reported micromagnetic calculations. In these calculations, R^2 is limited to values near 0.98 even in the best of circumstances. In general, our calculations here exhibit much better statistics, simply because we have much more particles in our calculations (10^6 hysterons) than were used in the original micromagnetic work (1330 gains).

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